

$$V = \{ A, B, C \}$$

$$\mathcal{T} = \{ A \rightarrow B, B \rightarrow C \}$$

$$\mathcal{T} \models A \rightarrow C$$

$$\mathcal{T} \models \frac{A \ B \rightarrow A}{\cdot}$$

$$\checkmark \quad \mathcal{T} \models A \ B \rightarrow C$$

$$\checkmark \quad \frac{A \ B \rightarrow B}{B \rightarrow C} \} \Rightarrow A \ B \rightarrow C$$

A	B	C
a ₁	b ₁	c ₁

a ₂	b ₁	c ₂
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A → B	✓
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B → C	X
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$$V = \{ A_1, A_2, \dots, A_n \}$$

$$\mathcal{R} = \left\{ A_1 \rightarrow A_2, \dots, A_{n-1} \rightarrow A_n, \right. \\ \left. A_n \rightarrow A_1 \right\}$$

$$A \rightarrow B, A \rightarrow C$$

$$\equiv A \rightarrow BC$$

$\mathcal{F}, V, X, \quad X \subseteq V$

$X \subseteq X^+$

$\mathcal{F} = \{ \underline{W} \rightarrow \underline{U} \dots \}$

r : if $W \subseteq X$, then add
 U into X

apply r recursively, till
no new attribute can be
added into X .

$$F = \{ A \rightarrow B, \textcircled{B} \rightarrow C \}$$

$$\frac{AB \rightarrow C}{}$$

$$\frac{(AB)^+}{} \xRightarrow{B \rightarrow C} \frac{ABC}{}$$



Super keys

$$A^+ = ABC$$

B is a key? AC a key?

$AB \rightarrow GH \quad ?$

$(AB)^+$

$\xRightarrow{AB \rightarrow E} ABE \xRightarrow[\begin{smallmatrix} E \rightarrow G \\ BE \rightarrow I \end{smallmatrix}]{\quad} ABEGI$

$\xRightarrow{GI \rightarrow H} ABEG \underline{I} \underline{H}$

$\approx AB \rightarrow GH$

$$\underline{AB \rightarrow GH ?}$$

$$\mathcal{R} : \{ AB \rightarrow E, BE \rightarrow I \\ E \rightarrow G, GI \rightarrow H \}$$

$$AB \rightarrow E \xrightarrow{A_2} \underline{AB \rightarrow BE}$$

$$\frac{A(B)}{BE \rightarrow I} \rightarrow \underline{AB \rightarrow I} \text{ (1)}$$

$$BE \rightarrow E (A_1) \xrightarrow[E \rightarrow G]{A_3} BE \rightarrow G$$

$$AB \rightarrow BE \xrightarrow{A_3} \underline{AB \rightarrow G} \text{ (2)}$$

$$\underline{\text{(1) + (2)}} \rightarrow AB \rightarrow GI$$

$$A_4 \xrightarrow[GI \rightarrow H]{A_3} AB \rightarrow H \text{ (3)}$$

$$\underline{\text{(2) + (3)}} \rightarrow AB \rightarrow GH$$

A4

$$V = \{A, B, C\}$$

$$F = \{A \rightarrow C, B \rightarrow C\}$$

A is a key?

AB

$$\{A \rightarrow B, B \rightarrow C\}$$

B is a key?

$$\{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$$

A, B, C

are the 3 keys.

$$F = \{AB \rightarrow C, C \rightarrow A\}$$

AB is a key.

① $(AB)^+ = ABC$

② AB is minimal.

A is not key $A^+ = A$

B is not key $B^+ = B$

BC is a key.

① $(BC)^+ = ABC$

② BC is minimal?

$$B^+ = B$$

$$C^+ = AC$$

AC a key?

① $(AC)^+ = AC$

AC is not a key.

$$V = \{ A, B, C, \textcircled{D} \}$$

$$T = \{ A \rightarrow B, B \rightarrow C, C \rightarrow A \}$$

$A\textcircled{D}$ is the only key.

AD, BD, CD

C, D, F

ABCDF